# CS 3630 Spring 2007
# Solution of Practice Test 1

## General

1. Why are legged robots especially useful in outdoor environments? Explain this from an energy consumption point of view.
   *Answer:* Since legged robots' only contact with the ground consists of discrete point contacts, they can work more efficiently in unstructured terrain. The energy expenditure of a wheeled robot in such terrain is much more substantial. In addition, a walking robot is capable of striding over a hole or chasm without going down and up or being trapped as wheeled robots would.

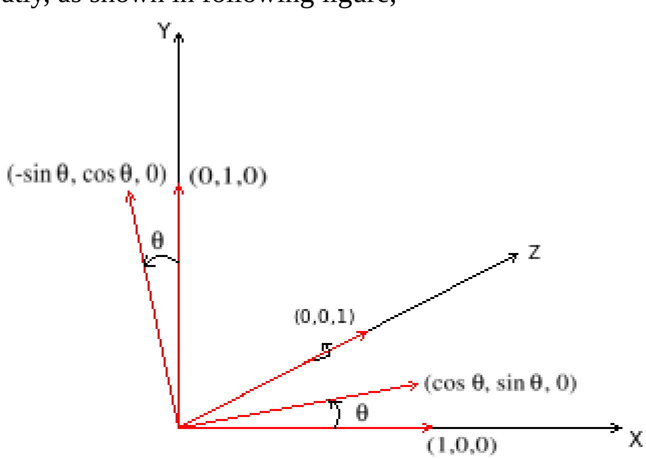
## Locomotion

1. What is the minimum number of wheels you need to possibly have a statically stable robot ?
   *Answer:* Two, where the center of gravity (CG) of the robot has to be *below* the wheel axis.

2. What is the steering principle used by the red, 4-wheeled ATRV robots in Arkin's mobile robot lab (and their smaller siblings in the BORG lab)?
   *Answer:* Skid Steering, the configuration of the 4 wheels does not allow for turning without slipping.

## Kinematics

1. Give the formula of the rotation matrix for yaw, i.e. rotation around the Z-axis, as a function of $\theta$, the angle by which we rotate. Show the derivation graphically.
   *Answer:* if we rotate three vectors $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ around the Z-axis separatly, as shown in following figure,



because $\begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} = R_{yaw}(\theta)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} = R_{yaw}(\theta)\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = R_{yaw}(\theta)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$,

then we get $R_{yaw}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

2. Derive the sliding constraint for a wheel at the origin of the robot frame, **aligned with the Y-axis** of the robot.

   (a) first, on the instantaneous velocity $\dot{\xi}_R$ in the robot frame$R$:
   *Answer:* Clearly, the sliding constraint in this case is that movement along the X-axis is not possible, i.e.,

   $$\dot{x_R} = 0$$

   which can be written as a constraint on $\dot{\xi}_R$ as follows:

   $$[\ 1 \quad 0 \quad 0\ ][\ \dot{x_R} \quad \dot{y_R} \quad \dot{\theta_R}\ ]^T = [\ 1 \quad 0 \quad 0\ ]\dot{\xi}_R = 0$$

   (b) second, on the instantaneous velocity $\dot{\xi}_I$ in the intertial (global/reference) frame $I$:
   *Answer:* Because $\dot{\xi}_R = R(\theta)\dot{\xi}_I$, we simply have (from (a)):

   $$[\ 1 \quad 0 \quad 0\ ]R(\theta)\dot{\xi}_I = 0$$

   or,

   $$[\ 1 \quad 0 \quad 0\ ]\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\dot{\xi}_I = 0$$

   Multiplying further, we simply have:

   $$[\ \cos\theta \quad \sin\theta \quad 0\ ]\dot{\xi}_I = [\ \cos\theta \quad \sin\theta \quad 0\ ][\ \dot{x_I} \quad \dot{y_I} \quad \dot{\theta_I}\ ]^T = 0$$

   which can also be written as

   $$\dot{x_I}\cos\theta + \dot{y_I}\sin\theta = 0$$